Spectral Centroid:

**A Spectral Feature for Signal Characterization**

Within the domain of digital signal processing (DSP), spectral centroid stands as a crucial quantitative descriptor used to characterize the frequency spectrum of a signal. It effectively pinpoints the center of mass within the spectrum, offering valuable insights into the distribution of constituent frequencies. Notably, spectral centroid exhibits a well-established perceptual correlation with the perceived brightness of an audio signal. It's also sometimes referred to as the "center of spectral mass."

**Mathematical Formulation:**

The calculation of spectral centroid leverages the concept of a weighted mean applied to the frequencies present in the signal. To achieve this, a Fourier transform is employed to transition the signal from the time domain to the frequency domain. The magnitudes associated with the individual frequency components then serve as weights within the following formula:

**Spectral Centroid = ∑ (f(n) \* x(n)) / ∑ x(n)**

**x (n):** represents the magnitude (weight) associated with frequency bin n.

**f(n):** represents the center frequency of frequency bin n.

**Theoretical Framework:**

1. **Discrete Fourier Transform (DFT):** We commence by leveraging the Discrete Fourier Transform (DFT). The DFT operates on a discrete signal x(n) residing in the time domain and transforms it into its constituent frequency components X(n) residing in the frequency domain. Each element X(n) encapsulates both the magnitude and phase information pertaining to a specific frequency n.
2. **Magnitude Spectrum: Our primary focus lies on the magnitude of the frequency components, denoted as |X(n)|. This quantity, obtained via the absolute value of X(n), reflects the strength of each frequency present within the signal.**
3. **Frequency Bins and Center Frequencies:** The DFT output X(n) manifests as an array wherein n represents discrete frequency bins. Each bin encompasses a specific range of frequencies. However, for spectral centroid calculations, we posit that each bin houses a single "center frequency" f(n). This center frequency approximates the central value within the frequency range encompassed by that bin.
4. **Weighted Mean Interpretation:** The spectral centroid essentially calculates a weighted mean of the center frequencies f(n). The weights employed in this calculation are the corresponding magnitudes |X(n)| retrieved from the frequency spectrum. A weighted mean inherently assigns greater significance to elements with higher weights.

**Optional Normalization:**

normalization of the spectral centroid by the signal's sampling frequency (fs) might be employed:

**normalized spectral centroid = spectral centroid / fs**

This normalization step scales the result to lie within the range of 0 and 0.5, where 0.5 represents the highest possible frequency (half the sampling frequency).

**Applications in Audio Signal Processing:**

**Spectral centroid finds numerous applications within the field of audio signal processing, particularly in tasks related to:**

* **Music Analysis and Genre Classification:** By analyzing the spectral centroid's temporal variations, researchers can glean valuable insights into the musical content. For instance, genres characterized by prominent percussive elements, like cymbal crashes, often exhibit corresponding spikes in the spectral centroid.
* **Timbre Description:** Spectral centroid plays a significant role in characterizing the overall tonal quality or "timbre" of a sound. Sounds with higher spectral centroids are generally perceived as brighter, while those with lower centroids tend to be perceived as warmer.
* **Audio Segmentation and Event Detection:** Variations in the spectral centroid can be leveraged to identify distinct segments within an audio clip. This characteristic proves beneficial in tasks such as segmenting speech from music or pinpointing specific sound events within an audio recording.